**Time Series Forecasting**

**Appendix: Discrete Fourier Transform**

The discrete Fourier transform is used to convert time series into the frequency domain. It can be used to try to pick out any periodicities/seasonalities in the series. The transform and its inverse are defined via:



as can be proved using:



If our sequence is real, so that xj = xj\*, and I imagine it would be, then there is an important symmetry to note. Consider the following manipulations:



So,



This indicates that only the first N/2 terms in the Fourier series are independent. If a sequence has a periodicity of T, then we could write it as something like:



where A is some complex constant. Filling this into the transform, we’d have:



which will be roughly zero due to oscillations, except if the exponent vanishes, which will happen when n = n\* = N/T. Conversely, if there is a peak in the amplitudes of the transform at say index n\*, then this corresponds to a period T = N/n\*. We’ll observe that from our analysis above, if there is a peak at n\* = N/T, there will also be a peak at n\* = N – N/T (as can verify if fill this alternate n\* into our analysis immediately above). In practice I’d say there is no point in looking at n for n > N/2, since the minimum period we can really pick out from the transform would be T ≥ 2, and this corresponds to n\* = N/T ≤ N/2.

**Example**

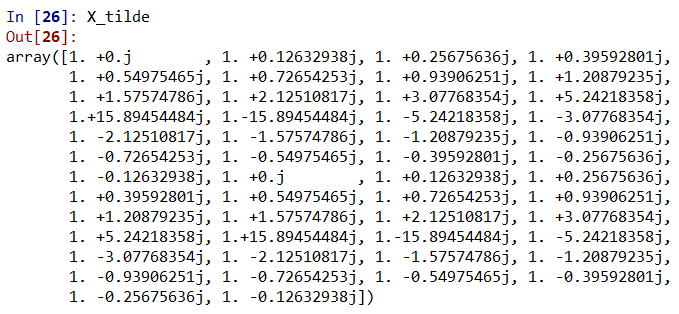
Okay, well let’s do an example. Consider the sequence: xn = 1, 0, -,1, 0, 1. Let’s work out the transform.



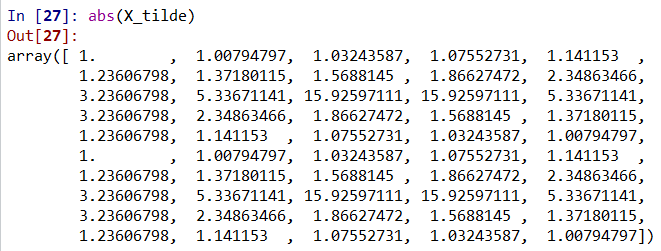
Numerically, these are:



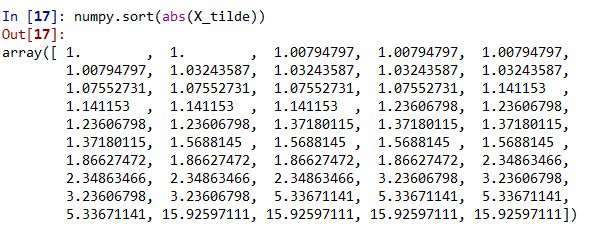
If we run this series out to N = 50, then we find, from numpy, the fft:



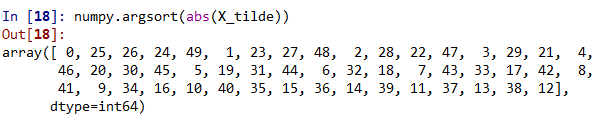
We’ll notice the symmetry that n\* = N-n. And if we take the magnitude (modulus) of these numbers, we find:



Can find the max values.



We see that 15 is the largest. Then we can find the indices corresponding to those values. We’ll ignore indices n > N/2 = 25, as per our discussion above.



So the indices are 12, 13. These correspond to periods:



These values correspond to the period T = 4, which would be n\* = 12.5. So this can be a good tool to find seasonalities. But that’s only if we eliminate any trends first. The Fourier transform gives useless results (for determining seasonality at least) when we have trends.